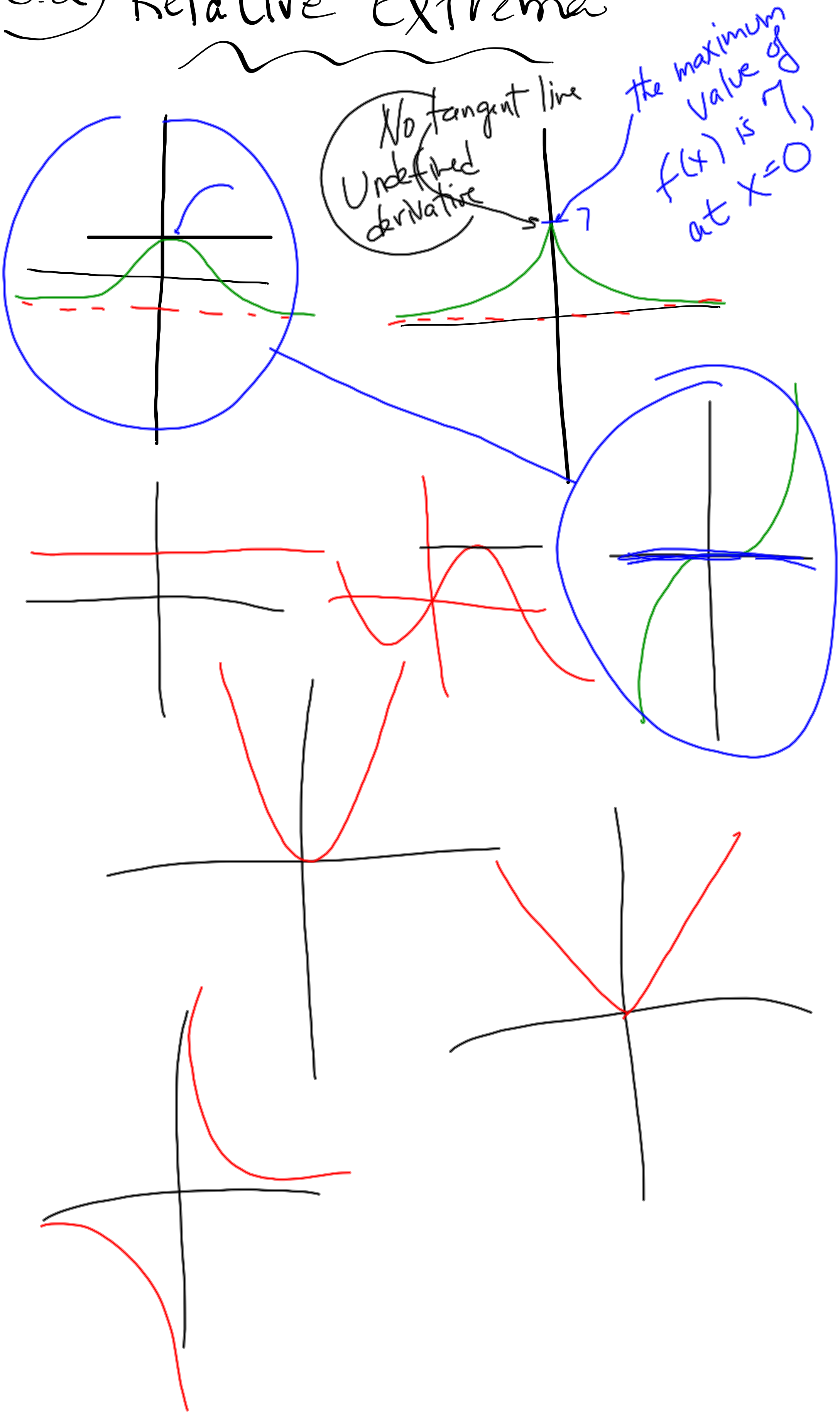
5.2) Relative Extrema



to find relative extremum

$y = x^2$

1) find $f'(x)$

$f'(x) = 2x$

2) solve $f'(x) = 0$
and $f'(x)$ undefined

CRITICAL VALUES
(numbers)

critical #s.

$2x = 0$
$\Rightarrow x = 0$

$2x$ never undefined

3*) evaluate critical numbers

first derivative test

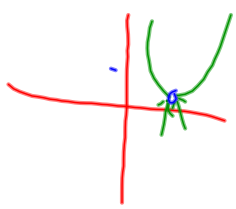
* create sign chart of $f'(x)$

$\frac{1}{x^2}$

* if $f'(x)$ goes from NEG to POS at a horizontal tangent then Relative MINIMUM
* if $f'(x)$: POS → NEG Relative Maximum
* if not nothing

second derivative test

* $f''(\text{critical}) > 0$
$\Rightarrow$ RELATIVE MINIMUM
* $f''(\text{critical}) < 0$
$\Rightarrow$ Relative MAXIMUM



DONE

7a) $f(x) = x^3 + 3x^2 - 9x + 1$

$f'(x) = 3x^2 + 6x - 9$

$= 3(x^2 + 2x - 3)$

$= 3(x+3)(x-1) = 0$

$x = 1, -3$

$+ + \;|\; - - - \;|\; + +$

$\quad -3 \qquad 1$

inc: $(-\infty, -3] \cup [1, \infty)$

dec: $[-3, 1]$

$\therefore$ rel max at $x = -3$

rel min at $x = +1$

$f''(x) = 6x + 6$

$f''(1) = 12 \;\therefore$ conc-up

$\therefore$ rel min

$f''(-3) = -18 + 6 = -12$

$\therefore$ conc-down

$\therefore$ rel max

$- - \;|\; + +$

$\quad -1$

C-up: $(-1, \infty)$

C-dn: $(-\infty, -1)$

pt of in: $x = -1$

b) $f(x) = x^4 - 6x^2 + 3$

$f'(x) = 4x^3 - 12x$

$= 4x(x^2 - 3)$

$= 4x(x-\sqrt{3})(x+\sqrt{3})$

$- - - \mid + + + \mid - - - \mid + + +$

$\quad -\sqrt{3} \qquad 0 \qquad \sqrt{3}$

$\therefore$ rel min @ $x = -\sqrt{3}, +\sqrt{3}$

rel max @ $x = 0$

inc: $[-\sqrt{3}, 0] \cup [\sqrt{3}, \infty)$

dec: $(-\infty, -\sqrt{3}] \cup [0, \sqrt{3}]$

$f''(x) = 12x^2 - 12$

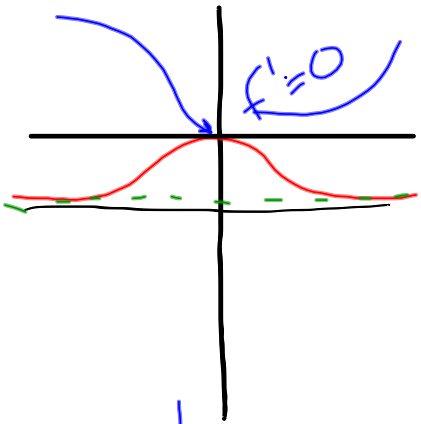
$f''(-\sqrt{3}) = 12(3) - 12 > 0$

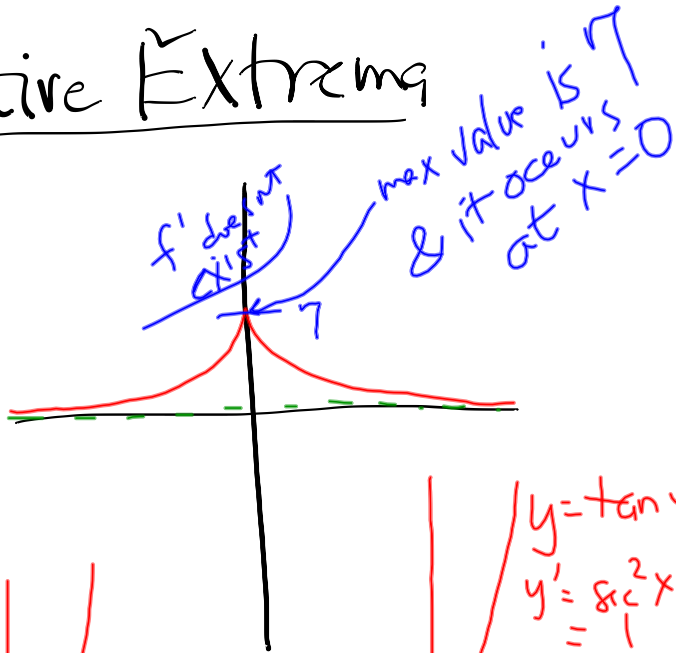
$f''(0) = -12 < 0$

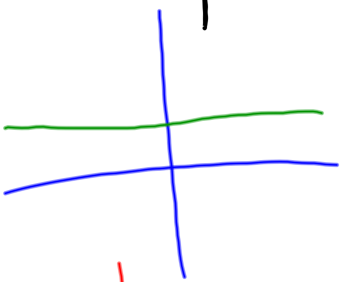
$f''(\sqrt{3}) = 12(3) - 12 > 0$

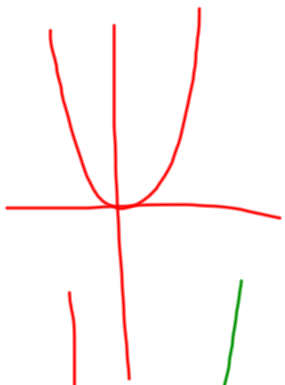
$\vdots$

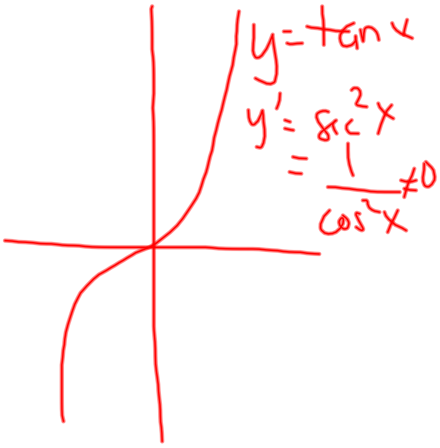
5.2 – Relative Extrema

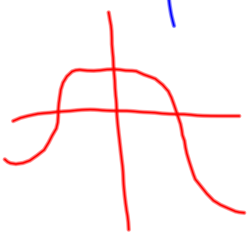


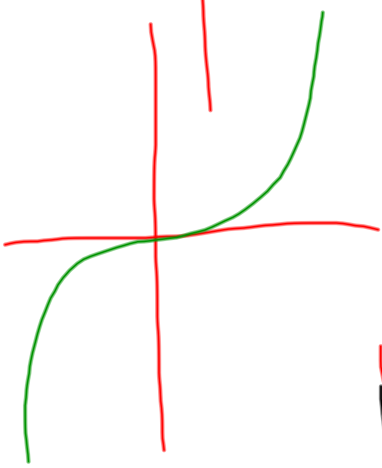


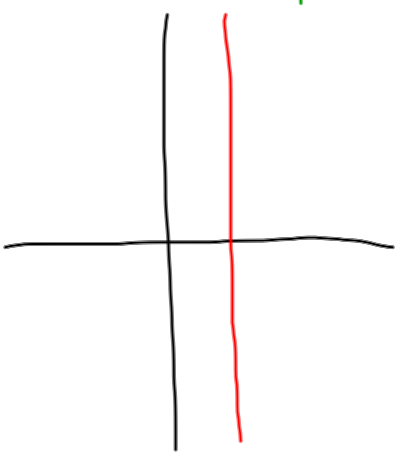


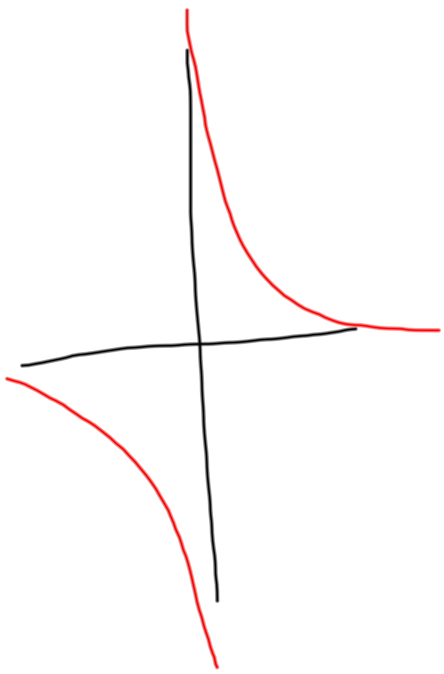












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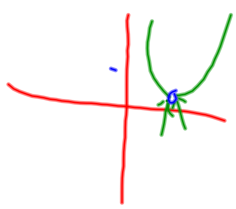

DONE

7a) $f(x) = x^3+3x^2-9x+1$

7b) $f(x) = x^4-6x^2-3$

$f'(x) = \frac{d}{dx}x^4 - \frac{d}{dx}6x^2 - \frac{d}{dx}3$

$= 4x^3-12x$

$4x(x^2-3) = 0$

$x = 0$

$x = \pm\sqrt{3}$

$-3 \quad -1 \quad 1 \quad 3$

$-\sqrt{3} \quad 0 \quad \sqrt{3}$

$- - - \; + + + + \; - - - - \; + + + +$

min max min

$f''(x) = \frac{d}{dx}4x^3 - \frac{d}{dx}12x$

$12x^2-12$

$12\cdot(-\sqrt{3})^2-12$

$12\cdot 3-12$

$24 = + = \text{Min}$

$12\cdot 0^2-12$

$0-12$

$-12 = - = \text{max}$

$12\cdot(\sqrt{3})^2-12$

$12\cdot 3-12$

$24 = + = \text{min}$